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This three-year project has focused on theoretical development and preliminary performance analysis for wavelet-based multiple access communication systems. Sets of orthonormal wavelet symbols suitable for asynchronous multiple access spread spectrum communications were developed.

Investigation of possible countermeasures to wavelet-based communication systems led to development of a cyclostationary signal detector for which detection thresholds corresponding to desired false-alarm probabilities can be determined analytically. This detector was applied against both traditionally modulated and low probability of detection signals, including wavelet modulated signals.

Some basic research on wavelet analysis performed in connection with this project led to a class of multidimensional generalizations of the wavelet transform.

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SCALE-BASED TECHNIQUES IN SECURE AND JAM-RESISTANT COMMUNICATION SYSTEMS

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Final Technical Report

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1 Introduction

This report summarizes the results obtained in a three-year investigation into the application of wavelet methods in modulation techniques for multi-user spread spectrum communication systems. The investigation was undertaken between November 1992 and January 1996.

2 Objectives

The objectives of this project remained essentially as outlined in the project proposal and previous status reports:

- **Task 1:** Development of and evaluation of wavelet symbols for multiple access spread spectrum communications.
- **Task 2:** Evaluation of communication system performance.
- **Task 3:** Investigation of possible countermeasures to wavelet-based spread spectrum.

In addition to addressing these objectives, some basic research on wavelet analysis was performed in connection with the project which led to a class of multidimensional generalizations of the wavelet transform.

3 Status of Effort

The project was completed in January, 1996 with the benefit of a two-month extension of the original three-year time line.

Task 1 led to development and analysis of a class of bandlimited orthonormal wavelet sets having properties well suited for use with the wavelet-based scale-division multiple access communication scheme around which much of this project was based.

Crosstalk effects in scale-division multiple access under asynchronous and distributed operation were studied under Task 2. Other performance questions originally planned to be addressed under this Task were de-emphasized in later stages of the project in view of published work on performance of wavelet-based communication protocols by other researchers.

Task 3 led to introduction and evaluation of an algorithm for detection of cyclostationary signals. This detector was used to evaluate of the detectability of scale-division multiple access signals by cyclostationary methods.

4 Accomplishments / New Findings

At the inception of this project, only a handful of researchers had looked into the possibility of using wavelets [8, 44, 49, 50] or fractal signals [57, 58] in the modulation of communication signals. The research group at Arizona State University was the first to investigate multiple access issues in this context [8]. There are now over forty published papers in this area [2, 5, 9-13, 17-21, 24-30, 33-36, 38-45, 47, 49-50, 53, 55-59].

Work under this effort focused on wavelet-based multiple-access techniques in which digital communication signals are encoded on orthogonal wavelet sets, as was introduced in [8] and described in the original proposal. Highlights of the results obtained in this project are summarized below.

4.1 Orthonormal Wavelet Symbols

Scale-division multiple access (SDMA) refers to a multiple-access communication scheme in which the orthogonal symbols are obtained by dilation and time shifting of a single prototype wavelet. The various users' messages are separated into channels based on the scale of dilation of the wavelet symbol on which their messages are encoded – hence the term SDMA [8, 10, 52]. As shown in these references, the spectral structure of a transmitted SDMA signal depends on the choice of the prototype wavelet symbol in a straightforward way.

The goals in construction of wavelets for use as symbols in SDMA are different from those encountered in designing wavelets for other applications. For use in time-frequency analysis of signals, for example, it is generally desirable for wavelets to have their energy simultaneously concentrated in both the time and frequency domains in order to provide signal analysis that is localized in the phase plane. In spread spectrum applications, however, wavelets should be compactly supported or highly concentrated in the time domain while having broadband frequency structure.

In general, SDMA relies on exact synchronization of all transmitters in the network to maintain orthogonality of the signals and avoid channel crosstalk. Even if this is achieved, time delays introduced by propagation through the transmission medium will cause the received signals to be non-orthogonal. This issue may be addressed by requiring that the channels in the network use symbols with Fourier transforms having disjoint supports. In this project, orthonormal wavelet bases generated by mother wavelets whose frequency spectra are supported in several disjoint bands were constructed.

The simplest example of a dyadic orthonormal basis of bandlimited wavelets is generated by the

mother wavelet defined in the frequency domain by

$$\hat{h}(\omega) = \begin{cases} 1 & \pi \le |\omega| < 2\pi \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

with $a_0 = 2$ and $b_0 = 1$. A related, but more complicated, example of an orthonormal basis of bandlimited wavelets was described by Mallat using the mother wavelet defined by

$$\hat{h}(\omega) = \begin{cases} 1 & |\omega| \in [\frac{4}{7}\pi, \pi) \cup [4\pi, \frac{32}{7}\pi) \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Examination of the relationship between these two wavelet bases gives insight about how to construct other bandlimited wavelet bases in which the dilations of the mother wavelet h do not overlap in the frequency domain.

In what follows, the connected components of the support of a mother wavelet's Fourier transform $\hat{h}$ will be called "slots." Throughout this section h will be a real-valued wavelet, which implies that $\hat{h}(-\omega) = \hat{h}^*(\omega)$ and hence the slots are symmetric about $\omega = 0$. Accordingly, when the values or support of $\hat{h}$ are specified on the positive frequency axis, they will be assumed to be given on the negative frequency axis by symmetry. The class of wavelets to be constructed will be categorized according to the number of slots they have in the positive frequency axis. The wavelet defined by (1) is thus a "one-slot" wavelet while the one defined by (2) is a "two-slot" wavelet. The following sections describe the construction of "n-slot" orthonormal wavelet bases.

Suppose the support S_0 of $\hat{h}$ is the union of n disjoint intervals

$$S_0 = [s_0, s_1] \cup [s_2, s_3] \cup \cdots \cup [s_{n-1}, s_n] \tag{3}$$

Then the support of $\hat{h}_m^k$ is

$$S_m = [s_0 2^{-m}, s_1 2^{-m}] \cup [s_2 2^{-m}, s_3 2^{-m}] \cup \cdots \cup [s_{n-1} 2^{-m}, s_n 2^{-m}]$$

If the measure of $S_m \cap S_{m'}$ is zero for all non-equal integers m and m', S_0 will be called an *orthogonal support*. In particular, an orthogonal support consisting of n disjoint intervals will be called an *n-slot orthogonal support*.

If $||h|| = 1$ and the support of $\hat{h}$ is orthogonal,

$$\left\langle h_m^n, h_{m'}^{n'} \right\rangle = \frac{1}{2\pi} \left\langle \hat{h}_m^n, \hat{h}_{m'}^{n'} \right\rangle = 0$$

for all $m \neq m'$. In this case, for the time-shifted and dilated replicates h_m^n of h to form an orthonormal set in L^2, it remains only to ensure that h_m^n and $h_{m'}^{n'}$ are orthonormal for $n \neq n'$ and all m.

This is simplified by the observation that

$$\langle h_m^n, h_m^{n'} \rangle = \langle h_0^n, h_0^{n'} \rangle$$

Thus, one must only verify that all of the time-shifted replicates of h are orthonormal at a single level of dilation.

4.1.1 Construction of Orthogonal Supports

Consider real numbers $0 < c_0 < \cdots < c_n = 2c_0$. Then

$$[c_0, 2c_0] = [c_0, c_1] \cup [c_1, c_2] \cup [c_2, c_3] \cup \cdots \cup [c_{n-1}, 2c_0]$$

Let $p_1, \ldots, p_n$ be distinct integers and dilate each sub-interval $[c_j, c_{j+1}]$ by $2^{p_{j+1}}$ to form a set

$$\begin{aligned} S_0 &= [c_0 2^{p_1}, c_1 2^{p_1}] \cup [c_1 2^{p_2}, c_2 2^{p_2}] \cup \cdots \cup [c_{n-1} 2^{p_n}, 2c_0 2^{p_n}] \\ &= [s_0, s_1] \cup [s_2, s_3] \cup \cdots \cup [s_{2n}, s_{2n-1}] \end{aligned}$$

Then S_0 is clearly an n-slot orthogonal support. Conversely, suppose S_0 is an n-slot orthogonal support of the form (3). Then it is straightforward to show that S_0 can be generated by the method just described with possibly some subintervals deleted.

4.1.2 Orthonormality of the Wavelets

Suppose $||h|| = 1$ and the support of $\hat{h}$ is orthogonal. Then $\{h_m^n\}$ is an orthonormal wavelet set if and only if $\langle h_0^n, h_0^{n'} \rangle = 0$ for all integers n and n' with $n \neq n'$. This is equivalent to

$$\begin{aligned} 0 &= \langle h_0^n, h_0^{n'} \rangle = \int_{\mathbb{R}} h(t-n) h^*(t-n')\, dt \\ &= \frac{1}{2\pi} \int_{\mathbb{R}} e^{-in\omega} \hat{h}(\omega) e^{in'\omega} \hat{h}^*(\omega)\, d\omega = \frac{1}{2\pi} \int_{\mathbb{R}} e^{i(n'-n)\omega} |\hat{h}(\omega)|^2\, d\omega \end{aligned}$$

for all $n \neq n'$. With $k \triangleq n - n' \neq 0$, this expression becomes

$$\int_{\mathbb{R}} e^{ik\omega} |\hat{h}(\omega)|^2\, d\omega = 0 \tag{4}$$

for any non-zero integer k. The following section presents some orthonormal wavelet sets corresponding to certain solutions of this equation.

4.1.3 Flat n-Slot Orthonormal Wavelets

The first n-slot orthonormal wavelets constructed were similar to the wavelets defined by (1) and (2) in that their Fourier transforms are constant on the n slots and zero elsewhere. These will be called *flat* n-slot wavelets. Construction of "non-flat" n-slot orthonormal wavelets is undertaken in the next section of this report.

Let

$$V = [c_0 2^{p_1}, c_1 2^{p_1}] \cup [c_1 2^{p_2}, c_2 2^{p_2}] \cup \cdots \cup [c_{n-1} 2^{p_n}, c_n 2^{p_n}]$$

where $0 < c_0 < \ldots < c_n = 2c_0$ and $p_1, p_2, \ldots, p_n$ are integers and define

$$\alpha = \left(\frac{1}{\pi} \sum_{j=1}^{n} 2^{p_j} (c_j - c_{j-1}) \right)^{-1/2}$$

Then if $\hat{h}(\omega) = \alpha$ for $\omega \in V$ and $\hat{h}(\omega) = 0$ otherwise, $||h|| = 1$ and equation (4) reduces to

$$\begin{aligned}
&\int_{\mathbb{R}} e^{ik\omega} |\hat{h}(\omega)|^2 \, d\omega \\
&= \alpha^2 \left(\int_{-c_1 2^{p_1}}^{-c_0 2^{p_1}} + \int_{c_0 2^{p_1}}^{c_1 2^{p_1}} + \int_{-c_2 2^{p_2}}^{-c_1 p_2} + \int_{c_1 2^{p_2}}^{c_2 2^{p_2}} + \cdots + \int_{-c_n 2^{p_n}}^{-c_{n-1} 2^{p_n}} + \int_{c_{n-1} 2^{p_n}}^{c_n 2^{p_n}} \right) e^{ik\omega} \, d\omega \\
&= 2\alpha^2 \left(\int_{c_0 2^{p_1}}^{c_1 2^{p_1}} + \int_{c_1 2^{p_2}}^{c_2 2^{p_2}} + \cdots + \int_{c_{n-1} 2^{p_n}}^{c_n 2^{p_n}} \right) \cos(k\omega) \, d\omega \\
&= \frac{2\alpha^2}{k} \{ -\sin(c_0 2^{p_1} k) + \sin(c_1 2^{p_1} k) - \sin(c_1 2^{p_2} k) + \sin(c_2 2^{p_2} k) - \\
&\qquad \cdots - \sin(c_{n-1} 2^{p_n} k) + \sin(c_n 2^{p_n} k) \} = 0
\end{aligned}$$

for all $k \neq 0$. This is equivalent to

$$-\sin(c_0 2^{p_1} k) + \sin(c_1 2^{p_1} k) - \sin(c_1 2^{p_2} k) + \sin(c_2 2^{p_2} k) - \cdots - \sin(c_{n-1} 2^{p_n} k) + \sin(c_n 2^{p_n} k) = 0 \quad (5)$$

for all $k \neq 0$. If $c_0 2^{p_1}$, $c_1 2^{p_1}$, $c_1 2^{p_2}$, $c_2 2^{p_2}$, ..., $c_{n-1} 2^{p_n}$, and $c_n 2^{p_n}$ satisfy equation (5), then so will $Mc_0 2^{p_1}$, $Mc_1 2^{p_1}$, $Mc_1 2^{p_2}$, $Mc_2 2^{p_2}$, ..., $Mc_{n-1} 2^{p_n}$, and $Mc_n 2^{p_n}$ for any natural number M. Thus, there exists a support of lowest dilation level which will be called the *mother support.* In other words, if the end points of the support

$$V = [c_0 2^{p_1}, c_1 2^{p_1}] \cup [c_1 2^{p_2}, c_2 2^{p_2}] \cup \cdots \cup [c_{n-1} 2^{p_n}, c_n 2^{p_n}]$$

satisfy (5) and the end points of the dilated support

$$[c_0 2^{p_1}/M, c_1 2^{p_1}/M] \cup [c_1 2^{p_2}/M, c_2 2^{p_2}/M] \cup \cdots \cup [c_{n-1} 2^{p_n}/M, c_n 2^{p_n}/M]$$

do not satisfy (5) for any $M \in \mathbb{N}$, then V is the mother support.

Example 1 In the one-slot case, the support consists of only one interval $[c_0 2^{p_1}, 2c_0 2^{p_1}]$ with $c_0 > 0$. Denoting $a = c_0 2^{p_1}$, equation (5) becomes $-\sin(ak) + \sin(2ak) = 0$, or

$$\sin(ak)[2\cos(ak) - 1] = 0$$

The only non-trivial solution of this equation that holds for any non-zero integer k is $a = m\pi$ with $m \in \mathbb{N}$. Hence, the support is $[m\pi, 2m\pi]$, the mother support is $[\pi, 2\pi]$, and an orthonormal wavelet basis obtained having the mother wavelet defined in equation (1).

Example 2 In the two-slot case, the cut points c_0 and c_1 and the dilation p should satisfy

$$-\sin(c_0 k) + \sin(c_1 k) - \sin(c_1 2^p k) + \sin(2c_0 2^p k) = 0$$

This equation can be decomposed into two equations (with loss of some solutions)

$$\begin{cases} \sin(2c_0 2^p k) - \sin(c_0 k) = 0 \\ \sin(c_1 2^p k) - \sin(c_1 k) = 0 \end{cases}$$

which can be solved simultaneously under the additional conditions $||h|| = 1$ and $\alpha = 1$ to yield

$$\begin{cases} c_0 = \frac{2^p}{2^{p+1}-1}\pi \\ c_1 = \pi \end{cases}$$

With $p = 0$ and $c_0 = \pi$, the wavelet obtained is the one defined in equation (1); $p = 2$ and $c_0 = \frac{4}{7}\pi$ yields the wavelet of equation (2). For $p = 1$ and $c_0 = \frac{2}{3}\pi$, a new bandlimited orthonormal mother wavelet defined by

$$\hat{h}(\omega) = \begin{cases} 1 & |\omega| \in [\frac{2}{3}\pi, \pi) \cup [2\pi, \frac{8}{3}\pi) \\ 0 & \text{otherwise} \end{cases}$$

is obtained.

4.1.4 Non-Flat n-Slot Orthonormal Wavelets

Denoting $\hat{\psi} = |\hat{h}|^2$, equation (4) becomes

$$\int_{\mathbb{R}} e^{ik\omega} |\hat{h}(\omega)|^2 \, d\omega = \int_{\mathbb{R}} \hat{\psi}(\omega) e^{ik\omega} \, d\omega = 2\pi\psi(k) = 0. \tag{6}$$

for all non-zero integers k. This formulation leads to the following:

Theorem 1 *Let $\hat{\psi}$ be a real-valued and non-negative function with orthogonal support. Further suppose $\int_{\mathbb{R}} \hat{\psi}(\omega)\, d\omega = 2\pi$ and $\psi(k) = 0$ for all non-zero integers k. Then with $|\hat{h}| = \sqrt{\hat{\psi}}$ and the phase of $\hat{h}$ chosen arbitrarily (except for conjugate symmetry), h is an orthonormal mother wavelet.*

In view of this result, construction of the desired non-flat wavelets hinges on finding functions ψ having the properties specified. Such functions may be constructed as follows.

Denote $\mathrm{sinc}(t) \triangleq \frac{\sin(\pi t)}{\pi t}$ and observe that $\mathrm{sinc}(k) = 0$ for all non-zero integers k. Its Fourier transform has values

$$\widehat{\mathrm{sinc}}(\omega) = \begin{cases} 1 & |\omega| < \pi \\ 0 & \text{otherwise} \end{cases}$$

Suppose $S = [c_0 2^{p_1}, c_1 2^{p_1}] \cup \cdots \cup [c_{n-1} 2^{p_n}, c_n 2^{p_n}]$ is an n-slot orthogonal support in which each slot has measure larger than 2π. Let $\hat{f}$ be any non-trivial L^2 function supported in $[c_0 2^{p_1} + \pi, c_1 2^{p_1} - \pi] \cup \cdots \cup [c_{n-1} 2^{p_n} + \pi, c_n 2^{p_n} - \pi]$ (figure 1) and whose convolution with $\widehat{\mathrm{sinc}}$ is non-negative. Then $\psi(t) \triangleq f(t)\,\mathrm{sinc}(t)$ is zero for all non-zero integer values of t, $\hat{\psi} = [\widehat{\mathrm{sinc}} * \hat{f}]$ is non-negative, and the support of $\hat{\psi}$ is orthogonal. Hence, an orthonormal mother wavelet h can be obtained by letting

$$|\hat{h}|^2 = \frac{2\pi \hat{\psi}}{\int_{\mathbb{R}} \hat{\psi}(\omega)\, d\omega}$$

and setting the phase of $\hat{h}$ arbitrarily.

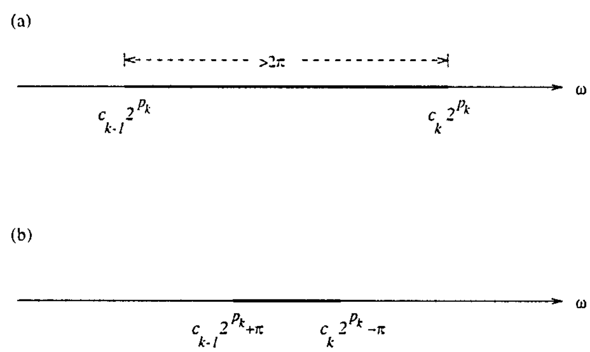


Figure 1: (a) Each interval in the original n-band orthogonal support S has length larger than 2π. (b) The support of $\hat{f}$ is formed by shrinking each interval in S by π on its left and right ends.

Note that h may be a dilated version of another orthonormal mother wavelet. If after rescaling, there exists some $N \in \mathbb{N}$, such that $h(2^{-N}t)$ satisfies equation (4), i.e.,

$$\int_{\mathbb{R}} e^{i\omega k} |\hat{h}(2^N \omega)|^2\, d\omega = 0$$

for all nonzero integers k, and

$$\int_{\mathbb{R}} e^{i\omega k} |h(2^{N+1}\omega)|^2\, dt \neq 0$$

for some nonzero integer k, then h' defined by $h'(t) = 2^{-N/2} h(2^{-N}t)$ is also an orthonormal mother wavelet.

The first part of the following result is a consequence of the above construction; the converse portion is proven in [10].

Theorem 2 *For any n-slot orthogonal support S_0, there exist orthonormal mother wavelets having support S_0. Conversely, any n-slot orthonormal mother wavelet can be constructed in the way discussed above.*

Example 3 Let $c_0 = \pi$, $p_1 = 1$, $c_1 = 7\pi/4$, and $c_2 = 2$. Then the support of $\hat{h}$ is $[\pi, 7\pi/4] \cup [7\pi, 8\pi]$. Expand this support by a factor of 4 to $[4\pi, 7\pi] \cup [28\pi, 32\pi]$ to make the length of each interval larger than 2π. Choose the support of $\hat{f}$ to be $[5\pi, 6\pi] \cup [29\pi, 31\pi]$ and generate $\hat{f}$ subject to the condition that $\hat{f} * \widehat{\text{sinc}}$ must be real and nonnegative; here define $\hat{f}$ by

$$\hat{f}(\omega) = \begin{cases} 1 & 5\pi < |\omega| \leq 6\pi \text{ and } 29\pi < |\omega| \leq 31\pi \\ 0 & \text{elsewhere} \end{cases}$$

Then

$$\hat{\psi}(\omega) = [\hat{f} * \widehat{\text{sinc}}](\omega)$$

and $\hat{\psi}$ is as shown in figure 2. Taking the square root of $\hat{\psi}$ as the absolute value of $\hat{h}$ and imposing zero phase, the orthonormal mother wavelet with Fourier transform depicted in figure 3 is obtained.

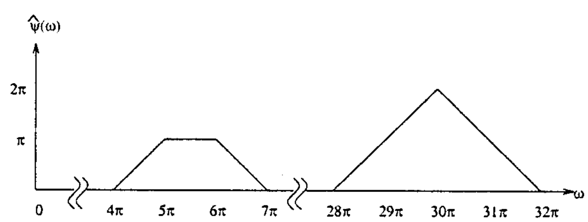


Figure 2: Convolution of $\hat{f}$ with the Fourier transform of a sinc function ensures $\psi(k) = 0$ for all non-zero integers k.

4.2 Orthonormal Wavelet Symbols with Frequency Overlap

If bandlimited wavelets are to be used as orthogonal symbols in a low-probability of exploitation (LPE) communication system, it may be desirable to have the bands overlap to avoid the possibility of an unintended receiver separating the channels using a relatively simple bank of bandpass filters. Following a technique developed by Suter and Oxley [46], bandlimited wavelet symbols have been

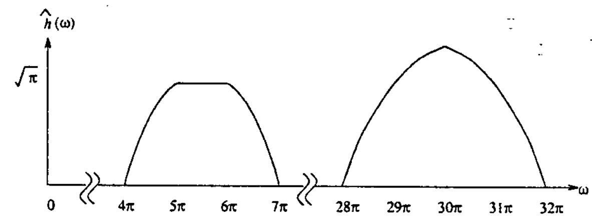


Figure 3: An orthogonal mother wavelet h is obtained as the inverse Fourier transform of the square root of $\hat{\psi}$.

developed which (i) generate orthonormal sets and (ii) whose replicates at different scales overlap in the frequency domain. The approach is summarized below.

The construction given in [46] assumes that the real line has been partitioned into disjoint intervals I_j, $j \in \mathbb{Z}$. If $\{f_{j,k} | k \in \mathbb{Z}\}$ is an orthonormal basis for $L^2(I_j)$, then $\{f_{j,k} | j, k \in \mathbb{Z}\}$ is an orthonormal basis for $L^2(\mathbb{R})$. A new orthonormal basis is constructed by extension of the functions $f_{j,k}$ off the interval I_j in a specific way, resulting in a basis in which the elements "overlap."

This construction is modified to the wavelet case to construct "overlapped" bandlimited orthonormal wavelet symbols as follows. Let h be a bandlimited wavelet having the properties:

1. The Fourier transform $\hat{h}$ of h is real-valued and even. The steps in the construction described below apply to the portion of $\hat{h}$ on the positive frequency axis and are assumed to also be applied to the portion of $\hat{h}$ on the negative frequency axis to maintain even symmetry.

2. The support of $\hat{h}$ is orthogonal in the sense introduced above; i.e., the portion on the positive frequency axis is of the form

$$[2^{d_1}\pi, 2^{d_1}a_1] \cup \cdots \cup [2^{d_n}a_{n-1}, 2^{d_n}2\pi]$$

   where $n \geq 1$, $\pi < a_1 < \cdots < a_{n-1} < 2\pi$, and $d_1, ..., d_n$ are integers. The construction given here also assumes $d_1 \neq 2d_n$.

For integers j and k, define

$$h_{j,k}(t) \quad = \quad 2^{-\frac{j}{2}} h\left(\frac{t - 2^j k}{2^j}\right) \tag{7}$$

and denote by $\hat{h}_{j,k}$ the Fourier transform of $h_{j,k}$.

Suppose $\{h_{j,k} | j, k \in \mathbb{Z}\}$ is an orthonormal basis of $L^2(\mathbb{R})$ (some examples of such bases are constructed above and others are described in [6]). With $\epsilon < \min\{(a_1 - \pi)/2, (2\pi - a_{n-1})/2\}$, define an extension $\tilde{\hat{h}}$ of $\hat{h}$ by

1. Constructing the odd extension of $\hat{h}$ about the point $2^{d_1}\pi$ into the interval $[2^{d_1}(\pi - \epsilon), 2^{d_1}\pi]$;

2. Constructing the even extension of $\hat{h}$ about the point $2^{d_n}2\pi$ into the interval $[2^{d_n}2\pi, 2^{d_n}(2\pi+\epsilon)]$; and

3. Repeating these steps on the negative frequency axis to preserve even symmetry.

An example is shown in figure 4 using Mallat's wavelet defined in equation (1).

Denote the extended wavelet just constructed by $\tilde{\hat{h}}$ and let $\hat{w}$ denote an frequency domain window function with support identical to that of $\tilde{\hat{h}}$ and with the amplitude normalization properties described in [46]. An example of $\hat{w}$ is depicted in figure 4. With u defined by

$$\hat{u}(\omega) = \hat{w}(\omega)\tilde{\hat{h}}(\omega) \tag{8}$$

the set $\{u_{j,k} | j, k \in \mathbb{Z}\}$ is shown to be an orthonormal wavelet basis of $L^2(\mathbb{R})$. Also, by construction, the dilated replicates of $\hat{u}$ have non-trivial overlap. Figure 5 shows a particular overlapped bandlimited orthonormal wavelet symbol $\hat{u}(\omega)$ constructed in the above example.

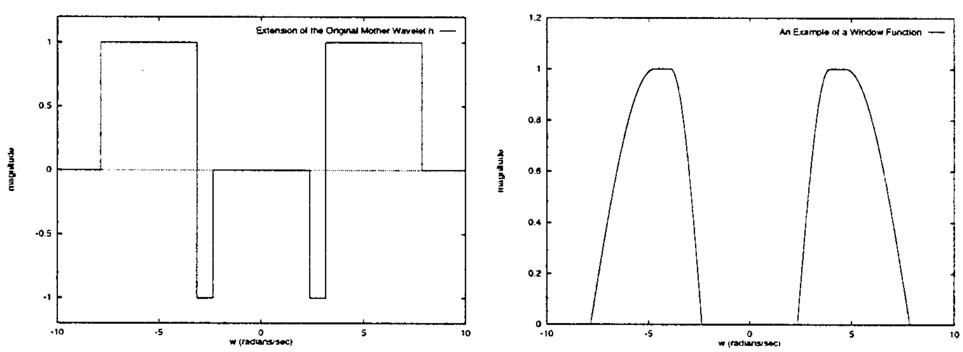


Figure 4: Initial frequency extension $\tilde{\hat{h}}(\omega)$ of the wavelet h (left) and the window function $\hat{w}(\omega)$ applied to it (right).

4.3 Countermeasures

A detector for cyclostationary signals was developed during this effort. This detector (depicted in figure 6) uses magnitude-squared coherence (MSC) estimation as a measure of the spectral correlation in a signal. The use of MSC estimation allows the statistical behavior of the detector in a noise-only environment to determined analytically and threshold values corresponding to desired false alarm probabilities to be computed [15, 16]. These capabilities represent a substantial improvement over similar cyclostationary detectors described in the research literature.

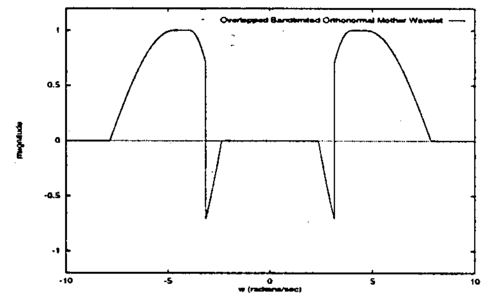


Figure 5: Example of an overlapped bandlimited orthonormal wavelet symbol $\hat{u}(\omega)$.

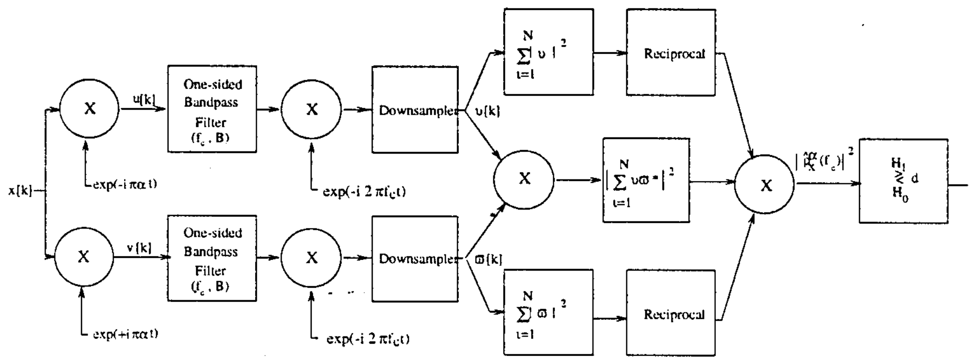


Figure 6: Discrete-time version of cyclostationary feature detector.

Receiver operating characteristic (ROC) curves for this detector against various types of cyclostationary communication signals have been determined by (i) using the analytical results to set detection a threshold for a particular false alarm probability, and then (ii) running computer simulations with signal present to estimate the corresponding detection probability. Performance results were obtained by applying the detector against standard communication signals (e.g., polar BPSK) and against some LPE-type communication signals, including direct sequence spread spectrum, transmitted reference spread spectrum, and SDMA signals. The case of SDMA signals was of particular interest in this project, and is discussed further below.

Consider an M-channel SDMA signal r described by

$$r(t) = \sum_{m=0}^{M-1} \sum_{k=-\infty}^{\infty} b_{m,k} \psi_k^m(t) \tag{9}$$

where $b_{m,k}$ represents the k^{th} bit on the m^{th} channel and ψ_k^m is k^{th} time-shifted replicate of the wavelet symbol ψ at dilation level m. The spectral correlation density (SCD) of such a SDMA signal

was found to be

$$S_x^\alpha(f) = \begin{cases} \dfrac{1}{T_0} \displaystyle\sum_{m=0}^{M-1} \sum_{k=-\infty}^{\infty} \hat{\psi}\left(\frac{f+\alpha/2}{2^m}\right) \hat{\psi}\left(\frac{f-\alpha/2}{2^m}\right)^* \delta\left(\alpha - \frac{n\,2^m}{T_0}\right) & \alpha = \dfrac{p}{T_0} \\ 0 & \alpha \neq \dfrac{p}{T_0} \end{cases} \tag{10}$$

where $\hat{\psi}$ is the Fourier transform of ψ, f is the spectral frequency, α is the cyclic frequency, and T_0 is the fundamental bit period (i.e., the time shift required for orthonormality of the wavelet symbols at the lowest scale of dilation). This expression shows that the SCD of a SDMA signals will be non-zero only at cyclic frequencies that are integer multiples of $1/T_0$. This behavior, which has also been verified empirically, indicates that SDMA signals exhibit cyclostationary characteristics that can possibly be exploited for the purpose of signal detection.

The cyclostationary feature detector was applied to detect the presence of a SDMA signal in noise. A six-channel SDMA signal was used as the input signal for the single-cycle detector. The wavelet symbol used in the SDMA signal was the Daubechies-4 wavelet. An initial analysis was performed to determine at which values of α the spectral correlation density would be nonzero. It was found that the SCD had a strong component at $\alpha = 2^6/T_0$ and $f = 0$. The single-cycle detector was set to operate at these values of f and α. The number N of independent samples used in the correlations performed by the single-cycle detector was set to 32. The bandwidth of the single-sided bandpass filters was set to $1.28/T_0$. Simulations were run at three different SNR's: 0dB. 3dB and 9dB. One thousand outputs were used in obtaining the empirical cumulative distribution for each SNR setting. The ROC curves obtained by this procedure are shown in figure 7. These results suggest that SDMA signals are vulnerable, at least to some extent, to detection by cyclostationary methods.

Another detector for polycyclic signals based on generalized coherence estimation [7] has been defined and its application against various types of communications waveforms, including SDMA, is currently under investigation.

4.4 Generalized Wavelet Transforms

Work in connection with this project has led to consideration of how the continuous wavelet transform can be generalized to higher dimensions. Some mathematical results arising from this work and a connection with image analysis are summarized in this section.

Several multidimensional generalizations of the one-dimensional wavelet transform are currently. In the most widely known (and obvious) generalization, one-dimensional transforms are applied

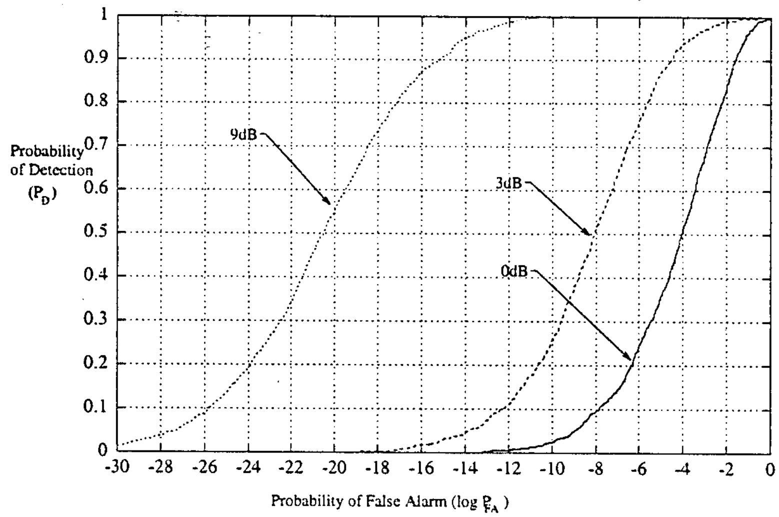


Figure 7: ROC curves for the single-cycle detector with a six-channel SDMA signal at its input.

separately in each orthogonal direction. In what follows, this will be referred to as the rectilinear wavelet transform; it is the method used by most wavelet-based image compression techniques. Another wavelet generalization is the two dimensional transform by Murenzi[37], literature on which can be more easily found in the paper by Antoine *et al.* [1]. Here, it will be called the circular wavelet transform, since it generalizes the dilations of the wavelet transform to the set of dilations and rotations on $\mathbb{R}^2$. Neither of these generalizations encompasses the other. Thus, a question of interest is whether there is a stronger generalization that encompasses both. The remainder of this section describes a generalized wavelet transform that includes as special cases the one-dimensional, rectilinear, and circular wavelet transforms. Among the multidimensional wavelet transforms that arise from this approach are several that appear to be unknown in engineering applications. The circular wavelet transform is shown to arise as a result of the relationship between $\mathbb{R}^2$ and $\mathbb{C}$ in the context of this generalization and is further shown to be strongly related to the cortex transform of Watson [54]. An implementation of the cortex transform from the circular wavelet transform is developed as an example of this relationship.

4.4.1 Theoretical development

Recall first the essentials of one-dimensional wavelet analysis, details of which can be found in [14]. With $a, b, x \in \mathbb{R}$, $h \in L^2(\mathbb{R})$, and $\hat{h}$ the Fourier transform of h, define

$$\left[\Delta^{(a,b)} h\right](x) = \frac{1}{\sqrt{a}} h\left(\frac{x-b}{a}\right)$$

and

$$C_h = \int_{\mathbb{R}} \frac{|\hat{h}(x)|^2}{|x|} dx.$$

If C_h is finite, h is admissible. In this case, the reconstruction theorem states that any $f \in L^2(\mathbb{R})$ can be reconstructed from its wavelet coefficients $\left\langle f, \Delta^{(a,b)} h \right\rangle$ by the formula

$$f = \frac{1}{C_h} \int_{\mathbb{R}} \int_{\mathbb{R}} \frac{1}{a^2} \left\langle f, \Delta^{(a,b)} h \right\rangle \Delta^{(a,b)} h \, db \, da.$$

The rectilinear wavelet transform generalizes the one-dimensional transform to a separable multidimensional transform by letting $a, b, x \in \mathbb{R}^n$, i.e.,

$$a = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} \quad b = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix},$$

with the following alterations in the definitions and theorem:

$$\left[\Delta^{(a,b)} h\right](x) = \frac{1}{\sqrt{a_1 \cdots a_n}} h \left(\begin{bmatrix} \frac{x_1 - b_1}{a_1} \\ \vdots \\ \frac{x_n - b_n}{a_n} \end{bmatrix} \right)$$

$$C_h = \int_{\mathbb{R}^n} \frac{|\hat{h}(x)|^2}{|x_1 \cdots x_n|} dx$$

$$f = \frac{1}{C_h} \int_{\mathbb{R}^n} \int_{\mathbb{R}^n} \frac{1}{a_1^2 \cdots a_n^2} \left\langle f, \Delta^{(a,b)} h \right\rangle \Delta^{(a,b)} h \, db \, da.$$

This generalization is equivalent to taking the one-dimensional wavelet transform successively in each orthogonal direction of the multidimensional space. For $n = 1$, the rectilinear wavelet transform reduces to the one-dimensional wavelet transform.

An alternative generalization of the one-dimensional wavelet transform to $\mathbb{R}^2$ is the circular wavelet transform [1, 37], for which $a \in \mathbb{R}$ is replaced with the pair $(a, \theta) \in \mathbb{R} \times [0, 2\pi)$ and $b \in \mathbb{R}$ is replaced by $b = [b_1, b_2]^{\mathrm{T}} \in \mathbb{R}^2$. Let r_θ denote the matrix which performs rotation in $\mathbb{R}^2$ by θ degrees. Then the definitions and theorem for circular wavelet analysis are

$$\left[\Delta^{(a,\theta,b)} h\right](x) = \frac{1}{a} h \left(a^{-1} r_{-\theta}(x - b) \right)$$

$$C_h = \int_{\mathbb{R}^2} \frac{|\hat{h}(x)|^2}{||x||^2} dx$$

$$f = \frac{1}{C_h} \int_{\mathbb{R}} \int_0^{2\pi} \int_{\mathbb{R}^2} \frac{1}{a^4} \left\langle f, \Delta^{(a,\theta,b)} h \right\rangle \Delta^{(a,\theta,b)} h \; a \, db \, d\theta \, da.$$

Clearly, the circular wavelet transform has a different character than the rectilinear wavelet transform on $\mathbb{R}^2$. In particular, it allows non-separable wavelets in two dimensions and replaces independent dilation in each dimension by rotation and dilation simultaneously in both dimensions. Despite their distinct characters, both the rectilinear and circular wavelet transforms arise as special cases of a more general multidimensional wavelet transform, the formulation of which is based on ring multiplications on $\mathbb{R}^2$.

Consider $\mathbb{R}^n$ as a vector space over the field $\mathbb{R}$. If a ring multiplication "$\circ$" is introduced to $\mathbb{R}^n$ so that $\mathbb{R}^n$ becomes an algebra under the multiplication then there is, for each $a \in \mathbb{R}^n$, a left regular representation $L_a : \mathbb{R}^n \to \mathbb{R}^n$ defined by $L_a x = a \circ x$. There is also an associated linear operator $S_x : \mathbb{R}^n \to \mathbb{R}^n$ defined by $S_x a = {L_a}^{\mathrm{T}} x$. Suppose the ring multiplication has the property that the set of elements with ring inverses are dense in $\mathbb{R}^n$. Then, with $h \in L^2(\mathbb{R}^n)$ and $a, b, x \in \mathbb{R}^n$, the following definitions and theorem form the mathematical foundation of a generalized multidimensional wavelet analysis:

$$\left[\Delta^{(a,b)} h\right](x) = \frac{1}{\sqrt{|\det L_a|}} h({L_a}^{-1}(x-b)),$$

$$C_h = \int_{\mathbb{R}^n} \frac{|\hat{h}(x)|^2}{|\det S_x|} dx.$$

$$f = \frac{1}{C_h} \int_{\mathbb{R}^n} \int_{\mathbb{R}^n} \frac{1}{|\det L_a|^2} \left\langle f, \Delta^{(a,b)} h \right\rangle \Delta^{(a,b)} h \; db \, da.$$

As examples of this, suppose the ring multiplication on $\mathbb{R}^n$ is defined by

$$\begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} \circ \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} a_1 b_1 \\ \vdots \\ a_n b_n \end{bmatrix}$$

then L_a and S_a are the diagonal matrices defined by $L_a = S_a = \mathrm{diag}(a_1, a_2, \ldots, a_n)$. This makes the generalized definitions and theorem equivalent to the definitions and theorem of rectilinear wavelet analysis. Since rectilinear wavelet analysis with $n = 1$ is the same as one-dimensional wavelet analysis, both one-dimensional wavelet analysis and rectilinear wavelet analysis arise as special cases of this formulation.

Similarly, circular wavelet analysis comes from the ring multiplication "$\circ$" on $\mathbb{R}^2$ defined by

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \circ \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} a_1 b_1 - a_2 b_2 \\ a_1 b_2 + a_2 b_1 \end{bmatrix}.$$

This ring multiplication is same as the multiplication of the complex numbers. In this case

$$L_a = \begin{bmatrix} a_1 & -a_2 \\ a_2 & a_1 \end{bmatrix} = |a| r_{\angle a},$$

$$S_x = \begin{bmatrix} x_1 & x_2 \\ x_2 & -x_1 \end{bmatrix},$$

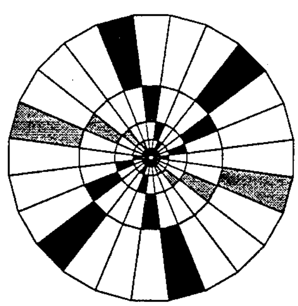
and the generalized definitions and reconstruction theorem become equivalent to the definitions and reconstruction theorem of circular wavelet analysis.

4.4.2 Connection to the cortex transform

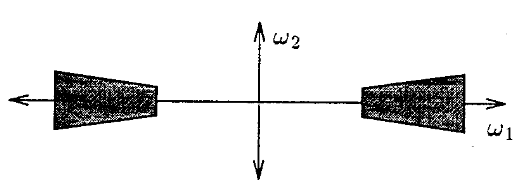
Circular wavelet analysis has fundamental variables of scale (which corresponds to logarithmic frequency), rotation, and two-dimensional positional shift. These correspond to the basic organization of Watson's cortex transform[54], which was devised as a means to process images in a similar way to the processing in the human visual cortex. Hence there is a relationship between the circular wavelet transform, the cortex transform, and image processing in the human visual cortex. It is shown here that this relationship can be strengthened by actually deriving the cortex transform from the circular wavelet transform.

The cortex transform was implemented by Watson by partitioning the frequency domain into separate angularly oriented regions at logarithmically spaced frequency steps, bandlimiting the image to each region, and sampling the bandlimited pieces as efficiently as possible using available techniques from sampling theory.

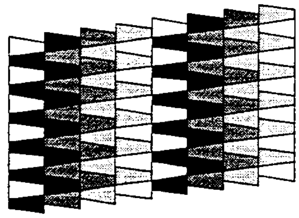
Following a similar procedure, the frequency domain can be divided according to the figure below, where zero frequency is in the center:



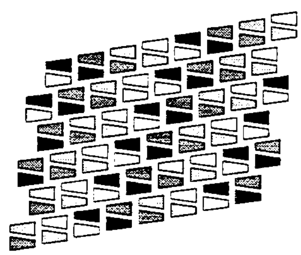
This division corresponds to a discrete sampling of the frequency and rotation variables of the circular wavelet transform. In order to bandlimit the image to a given region, the "mother wavelet" h is chosen so that its Fourier transform $\hat{h}$ is the characteristic function a *bin*, which consists of two opposing frequency regions as shown below:



Once the image is bandlimited to a bin, positional samples are used to represent the bandlimited function. These positional samples produce frequency replications. Aliasing is avoided by preventing these frequency replications from overlapping. An example of an optimal sampling for a bin is shown below.



This frequency replication pattern is according to theory the optimally efficient replication that avoids aliasing, but in practice aliasing is not avoided due to filter imperfections. Thus, it is often desirable to introduce a controlled amount of inefficiency in order to reduce aliasing. In this case, the frequency replication pattern looks more like this:



The efficiency of this sampling is controlled, so it is easy to make it more efficient than the fixed efficiency sampling proposed by Watson.

Preliminary results show that the transform adequately represents an image and reconstructs it from its representation (figure 8). In order to obtain these results, special consideration must be paid to preserving frequencies near zero, since the transform has a singularity there. It is also crucial that the image be of relatively large size (at least 256×256 pixels), since the processing assumes an image is continuous, and the violation of this assumption by a coarsely sampled image introduces significant artifacts.

5 Personnel Supported

Personnel contributing to this research effort were:

- D. Cochran, Principal Investigator
- S. Enserink, Research Assistant

Figure 8: The Leena image (left) and the result after it has been decomposed and reconstructed by the transform (right).

- S. Han, research Assistant
- R. Martin, Research Associate
- D. Sinno, Research Assistant
- C. Wei, Research Assistant

The research component of the following graduate degrees were sponsored or partially sponsored under this project:

1. C. Wei, Master of Science in Electrical Engineering, May 1993. Thesis: *Scale-Division Multiple Access*
2. S. Enserink, Master of Science in Electrical Engineering, May 1994. Thesis: *A Cyclostationary Feature Detector*
3. R. Martin, Doctor of Philosophy in Electrical Engineering, December 1994. Dissertation: *Multidimensional Wavelet Transforms and an Application to Image Processing.*

6 Related Publications

6a. Papers published

[6a-1] C. Wei and D. Cochran, "Construction of Discrete Orthogonal Wavelet Bases," *Record of the IEEE International Symposium on Information Theory*, San Antonio, January 1993.

[6a-2] C. Wei and D. Cochran, "Bandlimited Orthogonal Wavelet Symbols" (Invited paper), *Proceedings of the 27th Annual Asilomar Conference on Signals, Systems, and Computers*, Pacific Grove, California, November 1993.

[6a-3] C. Wei and D. Cochran, "A Wavelet-Based Multiple Access Spread Spectrum Modulation Scheme," *Proceedings of the IEEE International Phoenix Conference on Computers and Communications*, April 1994.

[6a-4] D. Cochran and C. Wei, "Wavelet-based spread spectrum communications," (Invited paper) *Proceedings of the IEEE Dual-Use Technologies and Applications Conference*, vol. II, pp. 392-401, Utica NY, May 1994.

[6a-5] S. Enserink and D. Cochran, "A Cyclostationary Feature Detector," *Proceedings of the 28th Annual Asilomar Conference on Signals, Systems, and Computers*, Pacific Grove, California, October 1994.

[6a-6] R. Martin and D. Cochran, "Generalized Wavelet Transforms and Cortical Filtering," *Proceedings of the 28th Annual Asilomar Conference on Signals, Systems, and Computers*. Pacific Grove, California, October 1994.

[6a-7] D. Cochran, "Review of Cyclostationarity in Communications and Signal Processing," (Invited book review) *Proceedings of the IEEE*, vol. 82, no. 10, October 1994.

[6a-8] S. Han and D. Cochran, "Orthonormal Bases of Bandlimited Wavelets with Frequency Overlap," *Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing*, vol. 3, pp. 1541-1543, Detroit, May 1995.

[6a-9] S. Enserink and D. Cochran, "On Detection of Cyclostationary Signals," *Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing*, vol. 3, pp. 2004-2007, Detroit, May 1995.

[6a-10] D. Cochran, H. Gish, and D. Sinno, "A Geometric Approach to Multiple-Channel Signal Detection," *IEEE Transactions on Signal Processing*, vol. SP-43(9), September 1995.

6b. Papers accepted for publication

[6b-1] D. Cochran and J.J. Clark, "Time-Warped Bandlimited Functions: Sampling, Bandlimitedness, and Uniqueness of Representation," To appear in: *SIAM Journal on Applied Mathematics.*

[6b-2] A. Clausen and D. Cochran, "An Invariance Property of the Generalized Coherence Estimate," To appear in: *IEEE Transactions on Signal Processing.*

6c. Recent submissions

[6c-1] A. Clausen and D. Cochran, "Generalized Coherence: Invariance of Estimate to Distribution of One Channel," Submitted to the 30th Asilomar Conference on Signals, Systems, and Computers, November 1996.

[6c-2] L.L. Scharf and D. Cochran, "On the Distribution of Linear Prediction Error," Submitted to the 30th Asilomar Conference on Signals, Systems, and Computers, November 1996.

[6c-3] J.Q. Trelewicz and D. Cochran, "A Sampling Approach to Bandlimited Instantaneously Companded Signals," Submitted to the 30th Asilomar Conference on Signals, Systems, and Computers, November 1996.

6d. Other related publications

[6d-1] D. Cochran, "Review of *Cyclostationarity in Communications and Signal Processing*," (Invited review) in *IEEE Communications Magazine*, vol. 32, no. 4, April 1994. Also in *Proceedings of the IEEE*, October 1994.

7 Interactions/Transitions

7a. Meetings, Conferences, and Seminars

Numerous university and industrial colloquium presentations have been presented on the research performed in this project. These include:

- D. Cochran presented a talk entitled "Wavelet-Based Multiple-Access Spread Spectrum" in the Motorola Government Systems and Technologies Group Signal Processing Series [16 attendees] in March 1993.

- D. Cochran presented a talk entitled "Detection of Cyclostationary and Polycyclostationary Signals" at the DSP group meeting at MIT [17 attendees] in October 1994. During the same visit, he met with A. Willsky, C. Karl, and R. Learned to exchange ideas on wavelet and wavelet-packet methods in spread spectrum.

- D. Cochran presented a talk entitled "Multidimensional Wavelet Generalizations and Watson's Cortex Transform" in the Vision Seminar at Harvard [25 attendees] in September 1994.

- D. Cochran presented a talk entitled "Wavelet Applications in Communications" in a seminar

sponsored jointly by the DSP and Communications groups at Georgia Tech [45 attendees] in October 1994.

In addition, D. Cochran participated in the following conference activities related to the topic of this project:

- He organized and chaired an invited session entitled "Applications of Wavelets and Chaotic Signals in Communications" at the Asilomar Conference on Signals, Systems, and Computers in November 1993. Participants included R. Orr (Atlantic Aerospace), S. Isabelle (MIT), M. Motamed and A. Zakhor (UC Berkeley), C. Wei (Arizona State University), and J.S. Goldstein (USAF and Georgia Tech).

- He was a keynote speaker (with J.J. Benedetto, M.V. Wickerhauser, and W. Sweldens) at a wavelet applications workshop in Melbourne, Australia in February 1995. The topic of this presentation was wavelet applications in communication systems.

- He is scheduled to be plenary speaker (with L. Cohen) at the ANZIIS-96 Conference in November 1996.

7b. Consultive and Advisory Functions

During this project, D. Cochran consulted informally with J. Stephens at Wright Laboratory and with Capt. J.S. Goldstein and Dr. A. Lindsey of Rome Laboratory on applications of wavelets and cyclostationary signal processing in covert communications and countermeasures. He has also been collaborating with the Centre for Signal and Information Processing Research (CSSIP) and the Defence Science and Technology Organization (DSTO) in Australia on wavelet applications in communications and other topics.

7c. Transitions

None.

8 New Discoveries

No inventions or patent disclosures have resulted from this research program.

9 Honors/Awards

None.

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